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OPTIMAL STAGING AND SCHEDULING IN AIRLIFT OPERATIONS

Sherwood W. Samn, Ph.D.

September 1981

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USAF SCHOOL OF AEROSPACE MEDICINE
Aerospace Medical Division (AFSC)
Brooks Air Force Base, Texas 78235

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This report shows that to achieve maximum aircraft utilization in airlift operations, optimal scheduling and staging can be formulated as a generalized network problem and solved using mixed-integer programming techniques.

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OPTIMAL STAGING AND SCHEDULING IN AIRLIFT OPERATIONS

INTRODUCTION

The staging problem in airlift operations is a generalized resource allocation problem of achieving maximum aircraft utilization by optimally staging aircrews at different airbases at the start of an airlift. The problem is inherently dynamic in nature and involves time delays. In this paper we will demonstrate how to model a simplified version of this problem as a time-expanded graph [1] and solve the resulting mixed-integer programming problem. The basic approach employed here is a generalization of that presented by Ford and Fulkerson [1] and Gearhart [2]. In particular, we address the problem of multiple routes, each of which may form a nonsimple loop. We also address the related problem of ordering, or prioritizing, routes in a linear programming environment.

THE PROBLEM

The basic system under consideration consists of P aircraft, C aircrews, and B airbases: B(1), B(2), ..., B(B). B(1) is the home base. This is where all the P aircraft are initially located and where all routes start and end. A route is any finite sequence of airbases

$$B(n_1),\ B(n_2),\ \ldots,\ B(n_k).$$

Here any consecutive pair $(B(n_i), B(n_{i+1}))$ is called a leg of the route. The only restrictions on any route are that the first and last stops should be the home base and that no other stop should be there. Note that not all airbases need to be included in a route and that an airbase may appear more than once in the same route.

A mission is a route with the associated attributes of aircraft and starting time. The number of routes in our system is fixed, but the number of missions can vary since more than one mission can follow the same route. The number of missions depends on the duration of the airlift operations under consideration.

Each mission is flown by a single aircraft; however, it is generally flown by more than one aircrew, each responsible for one leg of the mission. An aircrew is required to rest for a certain period of time after each leg of a mission, but the mission may be continued whenever any rested aircrew is available. If a rested aircrew is unavailable at an airbase B(n) to replace another (incoming) aircrew at the end of the leg (B(m),B(n)), the aircraft, and hence the mission in question, will have to be delayed until a rested aircrew does become available. In this simplified model, we have chosen to ignore a few restrictions both on the aircraft and the aircrews; some are ignored to keep the model simple, and others because their inclusion would render the problem unsolvable by reasonable analytical methods. Typical of

the latter is the limitation on the total flying hours of the aircrews during any 30-day period; a model including this restriction would have to include a running total of the flying hours of each aircrew. For a simulation model, this can be handled quite easily (for example, using SIMSCRIPT, although not Q-GERT nor GPSS); but for an analytic optimization model, this restriction is impractical to include.

The problem that we are considering here can be stated as follows: Given C aircrews, P aircraft, R routes, an ordered sequence of missions (to be elaborated later), and the duration of the airlift (time period T), how should the aircrews be initially distributed among the bases so that the flying hours by all aircraft are maximized during the airlift? The ordered sequence of missions alluded to is implicitly given by specifying the proportion, p(i) (i=1, ...,R), of the use of each route.

THE TIME-EXPANDED GRAPH FORMULATION

The basic idea as suggested by Gearhart [2] is to model the system as a time-expanded graph [1]. By assuming that all changes in the system take place at integer multiples of a basic time unit, which we will take as 1 for simplicity, we can discretize the time period (0,T) so that the system is fully described by its states at times 0, 1, 2, ..., T. Here T can be assumed to be an integer. As we shall see, our system can be represented by the graph G=(N,A), where N is a finite set of nodes and A is a finite set of arcs joining pairs of nodes. The graph is completely specified if we define all the nodes, all the arcs, all the flows along the arcs, and all the constraints. We will proceed to do this.

The Nodes

Four types of nodes comprise N: route, aircrew, schedule, and source nodes. We first define the set of route nodes associated with time n as R(n). The route nodes represent airbases B(1), B(2), ..., B(B), and possibly copies of them--depending on the route structure in the system. Specifically, suppose the system has R routes and that the i-th route has L(i) legs; then R(n) contains exactly L(1) + L(2) + ... + L(R) - R + 1 route nodes, each of which (with one exception) corresponds to a unique stop in a unique route. The same airbase may have to be represented by more than one node to ensure that aircraft will adhere to their routes. If airbase B(i) is the j-th stop in the r-th route, the node representing B(i) at time n will be B(i,n,r,j). The exception to a route node being a unique stop in a unique route is the node for home base, B(1). At time n, the common node for B(1) will be B(1,n), and home base will be identified by B(1,n,k,1) and by B(1,n,k,L(k)+1) (k=1, ...,R). The set of route nodes is the union of R(1), R(2), ..., and R(T).

The family of aircrew nodes consists of B nodes at each time n, each representing the aircrew pool at a specific airbase. These nodes will be denoted by C(i,n) (i =1,...,B; n=0,1,...,T).

The family of schedule nodes consists of a master schedule node, M, and R subsidiary schedule nodes, M(i,n) (i=1,...,R), for each time n (n=0,...,T).

The master schedule node is used to control the order in which the different mission types are generated, and the subsidiary schedule nodes are used as counters for the different types of missions that have been started on or before time n.

Finally the family of source nodes consists of exactly one node, S, representing the number of aircraft and aircrews in the system.

The Arcs

Arcs are ordered pairs of nodes. An arc is of interest only if a flow (along the arc) representing a meaningful quantity exists. Four types of arcs are in our system: arcs between route nodes, between aircrew nodes, between schedule nodes, and between source and other nodes. We will elaborate on these now.

Arcs Between Route Nodes--Three types of arcs connect the route nodes. These correspond to the three states an aircraft can be in--maintenance, delayed, and flying. The first type is of the form

$$(B(1,n),B(1,n+m)),$$

where m is the aircraft maintenance time at home base. The flow M(n) on one of these arcs represents the number of aircraft starting maintenance at time n. Recall that B(1,n,k,1) and B(1,n,k,L(k)+1) (k=1,...,R) both represent the same node (home base at time n) and are represented by B(1,n).

The second type of arc connecting the route nodes is of the form

$$(B(i,j,k,h),B(i,j+1,k,h)).$$

The flow D(i,n,k,h) on one of these arcs represents the number of aircraft being delayed (for example, waiting for a rested aircrew) at airbase B(i) at time n; the aircraft in question being at the h-th stop on the k-th route. In the special case in which i = 1 (home base), D(1,n,k,1) (k=1,...,R) represents the same flow and is denoted by D(1,n).

The last type of arc connecting the route nodes is of the form

$$(B(i,n,k,h),B(b(k,h+1),n+w(k,h),k,h+1));$$

here b(k,h) is the h-th stop on the k-th route, and w(k,h) is the flight time (an integer) between b(k,h) and b(k,h+1). The flow F(i,n,k,h) on one of these arcs represents the number of aircraft flying from b(k,h) to b(k,h+1), starting at time n and arriving at b(k,h+1) at time n + w(k,h).

Arcs Between Aircrew Nodes--Three types of arcs connect the aircrew nodes. These correspond to the three states an aircrew can be in--rest, delayed, and flying. The first type is of the form

$$(C(i,n),C(i,n+1)).$$

The flow d(i,n) on one of these arcs represents the number of aircrew being delayed (after being rested) for one unit of time at airbase B(i) starting at time n.

The second type of arc connecting the aircrew nodes is of the form

(C(i,n),C(i,n+r)).

The flow r(i,n) on one of these arcs represents the number of aircrew starting to rest (for r time units) at time n on airbase B(i).

The last type of arc connecting the aircrew nodes is of the form

(C(i,n),C(k,n+t(i,k))).

Here i is not equal to k, and t(i,k) represents the flight time between B(i) and B(k). We are assuming that all aircraft are of the same type, so the flight time between any two airbases can be assumed to be the same, independent of aircraft. The flow f(n,i,k) on one of these arcs represents the number of aircrews flying from airbase B(i) to B(k), starting at time n and arriving at B(k) at time n + t(i,k).

Arcs Between Schedule Nodes--Two types of arcs connect the schedule nodes. The first type is of the form

(M,M(i,n)).

These are arcs between the master schedule node and the subsidiary schedule nodes. The flow s(i,n) on one of these arcs represents the number of missions to be sent out on route i at time n. In our model, we assume s(i,n) is either one or zero.

The second type of arc connecting the schedule nodes is of the form

(M(i,n),M(i,n+1)).

As mentioned before, the subsidiary schedule nodes serve as counters for the flown missions of a given type. The flow S(i,n) on one of these arcs represents the total number of missions on route i that have been started on or before time N.

Arcs Between the Source Node and Other Nodes--Two types of arcs involve the source node, S. The first type is of the form

(S,B(1,0)).

The flow P along this arc represents the number of aircraft in the system. We assume all aircraft had their initial maintenance.

The second type of arc involving the source node is of the form

(S,C(i,0)).

The flow c(i) on one of these arcs represents the number of aircrews initially staged at the airbase B(i). We assume all aircrews are rested initially.

The Constraints

There are three types of constraints: integer, aircraft-aircrew, and scheduling. The integer constraints force the flows (variables) c(i), F(i,n,k,h), and s(i,n) to be integers. The aircraft-aircrew constraints force the number of aircrews flying out of any airbase to be equal to the number of aircraft flying out of that airbase. Finally, the schedule constraints force the right missions to be generated; a detailed discussion of this is given in Appendix A. One of the assumptions we are making on scheduling is that at any time n, the number s(n) of missions generated (regardless of type) is at most one. This assumption is made for purely technical reasons; without it we would have to deal with an increased number of equations. It does not affect practical application since the time unit can be made small enough for this assumption to be realistic.

THE MIXED-INTEGER PROGRAMMING FORMULATION

By balancing the flows at the nodes in our graph (except for the source node, S, and master node, M) and taking into consideration the constraints in the system, we can set up a mixed-integer programming model for our problem. To be more explicit, we will demonstrate this by working out an example. We assume there are three airbases (B=3)--B(1), B(2), and B(3)--and two routes (R=2):

Route 1: B(1), B(2), B(3), B(2), B(1)
Route 2: B(1), B(3), B(2), B(3), B(1).

We assume the flight time between airbase B(i) and B(j) is t(i,j), aircrew rest time is r, and aircraft maintenance time is m. We also assume the actual scheduling starts at time n = 0, so c(i) = d(i,0) (i=1,2,3). Then the equations and constraints describing this model are given as follows. (A list of the variables used and their definitions can be found in Appendix B.)

Source node:

$$d(1,0) + d(2,0) + d(3,0) = C$$
$$D(1,0) = P.$$

Route nodes (for n=1,2,...T):

$$M(n) = F(2,n-t(2,1),1,4) + F(3,n-t(3,1),2,4)$$
$$D(1,n) + F(1,n,1,1) + F(1,n,2,1) = D(n-1) + M(n-m)$$
$$D(2,n,1,2) + F(2,n,1,2) = D(2,n-1,1,2) + F(1,n-t(1,2),1,1)$$
$$D(3,n,1,3) + F(3,n,1,3) = D(3,n-1,1,3) + F(2,n-t(2,3),1,2)$$
$$D(2,n,1,4) + F(2,n,1,4) = D(2,n-1,1,4) + F(3,n-t(3,2),1,3)$$
$$D(3,n,2,2) + F(3,n,2,2) = D(3,n-1,2,2) + F(1,n-t(1,3),2,1)$$
$$D(2,n,2,3) + F(2,n,2,3) = D(2,n-1,2,3) + F(3,n-t(3,2),2,2)$$
$$D(3,n,2,4) + F(3,n,2,4) = D(3,n-1,2,4) + F(2,n-t(2,3),2,3).$$

Aircrew nodes (for $n=1,2,\ldots,T$):

$$
\begin{aligned}
f(n,1,3) + f(n,1,2) + d(1,n) &= d(1,n-1) + r(1,n-r)\\
r(1,n) &= f(n-t(2,1),2,1) + f(n-t(3,1),3,1)\\
f(n,2,1) + f(n,2,3) + d(2,n) &= d(2,n-1) + r(2,n-r)\\
r(2,n) &= f(n-t(1,2),1,2) + f(n-t(3,2),3,2)\\
f(n,3,1) + f(n,3,2) + d(3,n) &= d(3,n-1) + r(3,n-r)\\
r(3,n) &= f(n-t(1,3),1,3) + f(n-t(2,3),2,3).
\end{aligned}
$$

Schedule nodes (for $n=1,2,\ldots,T$):

$$
\begin{aligned}
S(n,1)-s(n,1) &= S(n-1,1)\\
S(n,2)-s(n,2) &= S(n-1,2).
\end{aligned}
$$

Aircraft-aircrew constraints (for $n=1,\ldots,T$):

$$
\begin{aligned}
f(n,1,2) &= F(1,n,1,1)\\
f(n,1,3) &= F(1,n,2,1)\\
f(n,2,1) &= F(2,n,2,3)\\
f(n,2,3) &= F(2,n,1,2) + F(2,n,2,3)\\
f(n,3,1) &= F(3,n,2,4)\\
f(n,3,2) &= F(3,n,1,3) + F(3,n,2,2).
\end{aligned}
$$

Schedule constraints (for $n=1,\ldots,T$, and W a sufficiently large number):

$$
\begin{aligned}
&F(1,n,1,1) = s(n,1)\\
&F(1,n,2,1) = s(n,2)\\
&2p(2)S(n,1) - 2p(1)S(n,2) + Ws(n,1) \le W + p(1) - p(2)\\
&2p(1)S(n,2) - 2p(2)S(n,1) + Ws(n,2) \le W + p(2) - p(1)\\
&s(n) - s(n,1) - s(n,2) = 0.
\end{aligned}
$$

Integer constraints: The integer variables are

$d(1,0)$, $d(2,0)$, and $d(3,0)$;

and the 0-1 integer variables are

$F(1,n,1,1)$, $F(2,n,1,2)$, $F(3,n,1,3)$, $F(2,n,1,4)$, $F(1,n,2,1)$, $F(3,n,2,2)$, $F(2,n,2,3)$, $F(3,n,2,4)$, and $s(n)$.

The objective function of this mixed-integer problem is the total flying hours by all aircraft and is given by

$$
\begin{aligned}
U = \sum_{n=1} \{ & F(1,n,1,1) + F(2,n,1,2) + F(3,n,1,3) + F(2,n,1,4)\\
& + F(1,n,2,1) + F(3,n,2,2) + F(1,n,2,3) + F(3,n,2,4)\}.
\end{aligned}
$$

Thus the problem of optimizing aircraft utilization reduces to maximizing U subject to all the constraints stated in this section.

RESULTS

The mixed-integer problem was solved on the UNIVAC 1108, using the mixed-integer programming (MIP) program in the Functional Mathematical Programming

System (FMPS) [3]. This computer program was developed by Bonner and Moore Associates, Inc. for Sperry Rand Corporation and is based on the ideas of Forrest et al. and Mitra [4,5]. Table 1 summarizes the results of several runs using different sets of parameters. In these runs, T = 12, P = 3, p(1) = 0.3, p(2) = 0.7, t(1,2) = 1.0, t(1,3) = 2.0, t(2,3) = 1.0, and m = 3.0. The only parameters that we changed from run to run were C, the number of aircrews, and r, the aircrew rest time. The solution to the mixed-integer problems includes aircraft utilization, U (defined here as the total number of basic time units flown by all aircraft during T basic time units), and the initial distribution of aircrews: d(1,0), d(2,0), and d(3,0) at airbases B(1), B(2), and B(3) respectively. Recall that the number of aircrews initially staged at an airbase is represented by c(i) = d(i,0) (i=1,2,3). (The quantity U shown in Table 1 is a slight overestimate of the actual aircraft utilization, since we have credited the complete flight time of a leg to an aircraft whenever it starts that leg; this induces overestimates on the flight times of all aircraft that are still flying at time T.) The dimension of the matrix that appeared in the MIP program in all cases is 330 rows (constraints) and 409 columns (variables), 117 of the latter being integers. The number of iterations (Iter) and CPU time shown in the table are dependent on the so-called cutoff value for that run; it is essentially a lower bound (provided by the user) for the objective function. A wisely chosen cutoff value can reduce dramatically the number of nodes to be searched in the branch-and-bound procedure, and hence the number of iterations and CPU time.

TABLE 1. RESULTS OF OPTIMIZATION

RUN	C	r	U	d(1,0)	d(2,0)	d(3,0)	Iter	CPU (min)
1	4	2	16	2	0	2	1195	9.98
2	4	3	*	*	*	*	*	*
3	4	4	14	3	0	1	2758	16.79
4	5	2	24	3	1	1	1136	9.16
5	5	3	20	3	1	1	1757	15.41
6	5	4	14	4	0	1	3150	18.98
7	6	2	24	3	1	2	2796	23.66
8	6	3	21	3	1	2	4213	38.46
9	6	4	18	3	1	2	3008	20.98

C--No. of aircrews in the system.
r--Aircrew rest time.
U--Total hours flown by all aircraft during the airlift.
d(1,0), d(2,0), d(3,0)--Initial distribution of aircrews at airbases.
Iter--No. of iterations.
*The Functional Mathematical Programming System [3] fails to yield a solution.

CONCLUSION

In this paper we have demonstrated the feasibility of using mixed-integer programming techniques to estimate the optimal staging policy in Air Force airlift operations. At present the method suffers from the dimensionality of the problem; the number of variables and constraints increases rapidly with the number of aircrews, the number of airbases, the number of routes, and most importantly, the duration of airlift operations. In an actual airlift problem, each of these parameters is generally tenfold larger than the one we have considered here. However, due to the sparsity and the structure of the matrix involved, there may be powerful (decomposition) techniques that can be used to circumvent this difficulty. This is an area for further research.

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APPENDIX A: MISSION SCHEDULING (ORDERING)

In this appendix, we will show how the proportion, p(i), of route usage can lead naturally to mission scheduling (ordering). Here we are given a finite sequence of proportion p(i) (i=1,...,R) so that

$$0 < p(i) \leq = 1, \text{ and}$$

$$p(1) + p(2) + \ldots + p(R) = 1$$

where R is the number of distinct routes. Let S(n) be the total number of missions that have been sent out before time n+1, and similarly let S(n,k) be the total number of missions of type k (k=1,...,R) that have been sent out before time n+1. Thus

$$S(n) = S(n,1) + S(n,2) + \ldots + S(n,R). \tag{A1}$$

Ideally, we would like to have S(n,k) = p(k)S(n) (k=1,...,R) for all n. But since S(n,k) and S(n) are integers, these conditions are seldom satisfied. We can view S(n,k)/p(k) (k=1,...,R) as R different estimates of S(n). Our problem is then to find a method for selecting the sequence of missions to be sent out so as to minimize the deviations (S(n,k)/p(k)-S(n)) (k=1,...,R) at each time *n*.

Note that from equation (A1) we have

$$S(n) = (p(1)+\ldots+p(R))$$
$$S(n) = S(n,k) + \ldots + S(n,R)$$

or

$$p(1)(S(n,1)/p(1)-S(n)) + \ldots + p(R)(S(n,R)/p(R)-S(n)) = 0.$$

That is, the average deviation is zero for all n. A criterion to use to insure small deviations (from S(n)) is that the average absolute deviations from S(n) be as small as possible for each n. Another criterion is that the average squared deviation should be minimized.

Minimization of Average Absolute Deviation

Assuming S(n,k) (k=1,...,R) has already been specified, the selection of the next type of mission should be based on minimizing the average absolute deviation (from S(n+1)) at time n+1; i.e.,

$$p(1)\left|S(n+1,1)/p(1) - S(n+1)\right| + p(2)\left|S(n+1,2)/p(2) - S(n+1)\right|$$
$$+ \ldots + p(R)\left|S(n+1,R)/p(R) - S(n+1)\right|. \tag{A2}$$

Note that if route i is picked at time n + 1, then

$$S(n+1,k) = S(n,k) + v(i,k)$$

and

$$S(n+1) = S(n) + 1.$$

Here $v(i,k) = 1$ if $i = k$, and 0 otherwise. Incidentally, $S(n)$ need not be n in general, since if an aircrew or an aircraft is not available, no mission can be sent out, and in that case $S(n+1) = S(n)$ and $S(n+1,k) = S(n,k)$ $(k=1, \ldots, R)$. The following lemma gives the selection rule based on minimizing expression (A2).

Lemma A. Let $Z(n+1,k)$ $(k=1,\ldots,R)$ be the average absolute deviation from $S(n+1)$--i.e., the value of expression (A2)--if a mission using the k-th route is chosen at time $n + 1$. And let $e(n+1,k) = S(n,k) - p(k)S(n+1)$; this is related to the difference between the actual and the expected number of missions using route k after $S(n+1)$ missions were scheduled. Then the problem of picking a mission type to minimize expression (A2), i.e.,

$$\min(Z(n+1,1),\ldots,Z(n+1,R)),$$

is equivalent (as far as picking the route) to

$$\min(e(n+1,1),\ldots, e(n+1,R)). \tag{A3}$$

Remarks:

1. The solution in general is not unique.

2. (A3) is equivalent to

$$\max(|e(n+1,k)|: \ e(n+1,k)<0) \tag{A4}$$

since
$$\begin{aligned} e(n+1,1)+ \ldots + e(n+1,R) &= (S(n,1)-p(1)S(n+1)) + \ldots + (S(n,R)-p(R)S(n+1)) \\ &= (S(n,1)-p(1)S(n)-p(1)) + \ldots + (S(n,R)-p(R)S(n)-p(R)) \\ &= -(p(1)+\ldots+p(R)) \\ &= -1, \end{aligned}$$

so $e(n+1,k)<0$ for at least one k.

3. In terms of (A4), the conclusion of the lemma is quite intuitive. As $e(n+1,k)$ is a measure of the difference between the actual and the expected number of type-k missions scheduled, the lemma says, first, that one should ignore those $e(n+1,k)$ that are positive; this makes sense, since $e(n+1,k)>0$ implies that type-k missions have been overscheduled. Secondly, it says that among underscheduled missions $(e(n+1,k)<0)$, the one most underscheduled should be chosen.

Proof of Lemma A. If a type-k mission is chosen, then the average absolute deviation from $S(n+1)$ is

$$\begin{aligned} Z(n+1,k) = p(1)\left|(S(n,1)+v(1,k))/p(1) - S(n+1)\right| + \ldots \\ +p(R)\left|(S(n,R)+v(R,k))/p(R) - S(n+1)\right|. \end{aligned}$$

Now $Z(n+1,k)$ can also be written as

$$
\begin{aligned}
Z(n+1,k) = p(1)\left|S(n,1)/p(1) - S(n+1)\right| + \ldots + p(R)\left|S(n,R)/p(R)\right. \\
\left. - S(n+1)\right| + p(k)\left|(S(n,k)+1)/p(k)-S(n+1)\right| \\
- p(k)\left|S(n,k)/p(k) - S(n+1)\right|.
\end{aligned}
$$

Ignoring the terms that do not depend on k (and thus do not contribute anything to the minimization process), the original minimization is equivalent to (as far as picking the route)

$$\min(|e(n+1,k) + 1| - |e(n+1,k)|) \qquad \text{(A5)}$$

where

$$e(n+1,k) = S(n,k) - p(k)S(n+1).$$

Now the function $f(x) = |x + 1| - |x|$ is nondecreasing (-1 if $x < -1$; $2x+1$ if x is between -1 and 0; and 1 if $x > 0$); hence (A5) is equivalent to

$$\min(e(n+1,1),e(n+1,2),\ldots,e(n+1,R)).$$

Minimization of Average Squared Deviation

Instead of minimizing (A2), here we minimize $p(1) \times (S(n,1)/p(1)-S(n))^2 + \ldots + p(r) \times (S(n,R)/p(r)-S(n))^2$. By a similar argument, we can show that the type-k mission to be selected at time $n + 1$ (if any) is one for which

$$(2S(n,j)+1)p_j \quad (j=1,\ldots,R)$$

is minimum. We have used this criterion in our sample problem.

APPENDIX B: NOMENCLATURE

A	The family of arcs in the graph, G, representing the airlift system.
B	The number of airbases in the system.
B(i)	The i-th airbase in the system. (B(i) is the home base.)
B(1,n)	A route node representing the home base at time n.
B(i,n,k,h)	A route node representing airbase B(i) at time n; this is the h-th stop on the k-th route.
C	The number of aircrews in the system.
c(i)	The number of aircrews initially staged at airbase B(i).
C(i,n)	An aircrew node representing airbase B(i) at time n.
D(1,n)	The number of aircraft delayed at home base at time n.
D(i,n,k,h)	The number of aircraft delayed at airbase B(i) at time n and on the h-th stop of the k-th route.
d(i,n)	The number of aircrews delayed at airbase B(i) at time n.
e(n,k)	The difference between the actual and expected number of missions of type k scheduled at time n.
F(i,n,k,h)	The number of aircraft leaving airbase B(i) at time n, where B(i) is the h-th stop on the k-th route.
f(n,i,k)	The number of aircrews leaving B(i) towards airbase B(k) at time n.
G	The graph representing the airlift system.
L(i)	The number of legs in the i-th route. There are L(i) + 1 stops in the i-th route, B(1) being the first and the last.
M	The master schedule node.
M(n)	The number of aircraft starting maintenance at time n.
M(i,n)	A subsidiary schedule node related to the i-th route at time n.
m	Aircraft maintenance time.
N	The family of nodes in the graph, G, representing the system.
P	The number of aircraft in the system.
p(i)	The proportional usage of route i.

R	The number of routes in the system.
R(n)	A set of route nodes related to airbases at time n.
r	The aircrew rest time.
r(i,n)	The number of aircrew resting at airbase B(i) at time n.
S	The source node in the graph, G, representing the system.
S(n)	The total number of missions sent out as of time n.
S(n,k)	The total number of type-k missions that were sent out as of time n.
s(n)	The total number of missions that are sent out at time n.
s(n,k)	The total number of type-k missions that are sent out at time n.
T	Duration of airlift being considered.
t(i,j)	Flight time between airbases B(i) and B(j).
U	Aircraft utilization; i.e., the total number of flying hours by all aircraft during time period T.
v(i,j)	A number equal to 1 when i = j and to 0 otherwise.
w(k,h)	Flight time between airbases b(k,h) and b(k,h+1).

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